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# Quasi-Linearization Method for State Estimation Problems

Anton B. Tmur, Energoavtomatika, Ltd., Russian Federation



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## ABSTRACT

The pipeline state estimation problem is posed and considered by the example of the simplest pipeline system. The mathematical model of the considered pipeline is formulated. It contains the simplified Navie-Stokes equations for the interior points of the pipe, and two boundary conditions for the fluid flow through the pump and valve. The main principles of the difference scheme for solving these model equations are described. The iterative quasi-linearization is proposed. The state estimation procedure is built on the basis of quasi-linearized model. Some results of numerical experiments are given.

## INTRODUCTION

In the past few years the state estimation problem for pipeline systems has become one of current importance. There are different approaches to this problem, using the online models. Most of them are built on the correct statement of boundary conditions [6]. This paper represents a new technique for online state estimation based on the pipeline model and a latest history of data, obtained from pressure sensors and flowmeters installed on the pipeline.

The point is that in spite of the fact that modern oil pipelines are equipped with high tech equipment the accidents and oil spills keep happen, damaging the environment. To prevent these accidents one should have clear understanding about what is going on inside the pipeline. That is why mathematical models are attracted. They are generally accepted as the cheapest way of the investigation. Using mathematical modeling one can model almost all dangerous situations that take place in the real pipeline. But this is not enough.

The pipeline when in use is the complex dynamical system, with many input and output signals. The input signals are the operator's commands, such as valve shutting, pump start, tank changeover, etc. Output signals are the measurements of pressure and flow in situ, using special pressure sensors and flowmeters. As the case may be the pipeline can reach different states, depending on what command the operator has input. The wrong command can entail serious consequences. For example, the wrong stop of the pump can entail pipeline breakdown and therefore oil spill. But if it was possible for operator to control the flow process in the pipeline using its' online model one would avoid the accident.

Mathematical model of a pipeline as a system with distributed parameters is represented with the hyperbolic system of partial equations and boundary conditions. To solve this system means to obtain the system state in the future time moments. But solving the hyperbolic system of partial equations is impossible without known initial state and boundary conditions at the next time moments. To know the state of the pipeline means to know the flow velocity and pressure distributions along the pipe. Despite the boundary conditions the initial state cannot be measured directly. Hence, one has to use identification methods to obtain the current state and to make a prediction on the basis of one.

Identification of the current state can be carried out on the basis of some measurements from the past, i.e. using the latest history of data, obtained from pressure sensors and flowmeters installed on the pipeline. One of the possible approaches to this problem is considered in this paper.

# MATHEMATICAL MODEL

### Pipeline layout

The pipeline system considered is shown in Fig. 1. It consists of a pipe, two tanks, a pump on the left bound and a valve on the right bound, which can be represented as a local resistance. As Fig. 1 shows, the calculation area is enclosed between the pump and the valve. The case where fluid is flowing from left to right is considered.

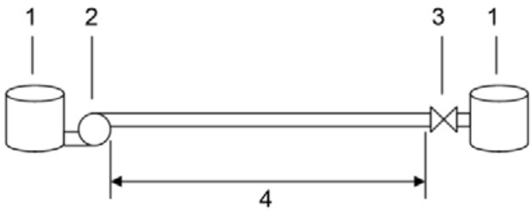



Fig. 1. Pipeline layout. 1 – tank, 2 – pump, 3 – valve, 4 – pipe, calculation area.

The necessary physical parameters of every pipeline element are supposed to be known. For example, if the pipe is mentioned, we assume that the pipe diameter, length and elevation are measured before and known. In the same way the pump characteristic coefficients, diameter of the tank and capacity of the valve are considered to be known.

### Model equations

We will consider the current in the pipeline as an isothermal flow of the poorly compressible fluid. Under these conditions the flow process can be described using the Navie-Stokes equations for the round section channel [1]:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} = 0, \quad (1)$$

$$\frac{\partial (\rho u)}{\partial t} + \frac{\partial \left(\rho u^2\right)}{\partial x} = -\rho g \sin \gamma - \frac{\partial p}{\partial x} - \lambda \frac{\rho u |u|}{2D}, \quad (2)$$

where ρ is density, u is flow velocity, p is pressure, t is time, x is space coordinate, g is gravitational acceleration, γ is pipe inclination, D is inner diameter, λ is hydraulic resistance coefficient. Equations (1) and (2) represent continuity equation and flow equation correspondingly. The first term in the right part of (2) accounts for the gravitational force. The second term accounts for the pressure difference and the third one represents the friction due to motion. The last third term has the empirical coefficient of hydraulic resistance λ.

There are a lot of formulas for calculating the hydraulic resistance coefficient λ. All of them have an empirical origin. In this paper the Blazius formula is used [5]:

$$\lambda = \frac{0.3164}{\sqrt[4]{\mathrm{Re}}}, \quad (3)$$

where Re is a Reynolds number. In general case the hydraulic resistance coefficient is dependent on the relative pipe roughness.

We use dependence for the adiabatic sound velocity in the fluid, as a closure equation (Glikman, 1986):

$$c^2 = \left(\frac{\partial p}{\partial \rho}\right)_{s=const}, \quad (4)$$

where c is sound velocity, s is entropy. Equation (4) helps in reducing of the system (1), (2) to the system of partial equations with the only two independent parameters: pressure and velocity. Thus, the state of the flow at any inner section of the pipeline can be characterized with these two parameters.

### Boundary conditions

As it is shown in Fig. 1, there are tanks on the both ends of the pipeline. Let us consider tanks with unchangable fluid level inside. This simple case can be easily reformed to the tank with changeable level. Both cases give us the information about the pressure at the point of entry. So, we will use the next condition for tanks' modeling:

$$p_{left} = const, \quad (5)$$

$$p_{right} = const, \quad (6)$$

where $p_{left}$ and $p_{right}$ are the pressures in the tanks on the left and right bounds correspondingly.

Provided that density and viscosity is constant, the centrifugal pump at the left end of the pipe can be modeled, using pump characteristic [2]:

$$\Delta p = AN_r^2 + BN_r u + Cu^2, \quad (7)$$

where $N_r$ is revolutions per minute, A, B, C are pump characteristic coefficients, Δp is pressure difference between pump's input and output:

$$\Delta p = p_{output} - p_{input}. \quad (8)$$

Provided the same condition about density and viscosity the flow through the valve can be expressed as [2]:

$$u = K_V \sqrt{|\Delta p|}, \quad (9)$$

where $K_V$ is carrying capacity of the valve, Δp is defined in (8).

### Grid generation and difference scheme

As it was mentioned above, the calculation area is bounded with the pump and valve. The above system of partial equations (1), (2) cannot be solved analytically. That is why one has to use numerical methods to get the solution. The regular grid generated for the further modelling is depicted in Fig. 3. It consists of M nodes in space and N nodes in time.

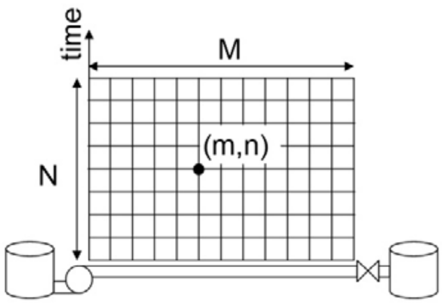



Fig. 3. The calculation grid.

Let this grid have a space step *h* and a time step *τ*.

Every node of this grid has unique couple of indexes m and n indicating its place in the grid. Indexes m and n represent the space and time indexes correspondingly. Pressure and velocity at every node are denoted as $p_m^n$ and $u_m^n$. That is the subscript denotes grid space coordinate and superscript denotes grid time coordinate.

As for a numerical method there are a lot of well-known difference schemes for solving hyperbolic system of partial equations on the regular grid. One may use any of it. In this paper the characteristics method [1] is used. This method applies restriction on the mesh.

It does not matter for identification method described below what difference scheme is used. There are only two important properties this scheme must have. First of all, it must be explicit. Secondly, the difference equations must include the parameters from two layers only. The last two conditions lead to ability of representing difference scheme as a rule, which set up a correspondence between parameters at time layers *n* and *n+1*.

### State vector

The current state at every time-layer n is characterized by $2 \cdot M$ parameters: velocities and pressures at M points. For the sake of simplicity the state vector z is introduced. This vector consists of M velocities and M pressures, written in order. Vector z has the following form:

$$\mathbf{z}_n = \left[u_1^n, u_2^n, \ldots u_M^n, p_1^n, p_2^n, \ldots, p_M^n\right]^T \quad (10)$$

Thus, at every time-layer n of the introduced grid the current regime is completely characterized with the vector $\mathbf{z}_n$. Then one can rewrite the difference scheme (see Section 2.4) as some vector-function ψ:

$$\mathbf{z}_{n+1} = \psi(\mathbf{z}_n). \quad (11)$$

This function can be easily made up on the basis of chosen difference scheme, provided that one satisfies the conditions described in Section 2.4.

The modeling of the transient processes should be carried out using the boundary conditions. It means that at every time step n one have to know the pump number of revolutions per second $N_r$, degree of the valve shutting $F_{valve}$ and pressure in the tanks $p_{left}, p_{right}$ ($F_{valve}$ presents implicitly in $K_V$, see Eq. (9)). That is, in the general case the dependence (11) has the form:

$$\mathbf{z}_{n+1} = \psi(\mathbf{z}_n, N_r, F_{valve}, p_{left}, p_{right}). \quad (12)$$

Again, for the sake of simplicity we will consider boundary conditions as known and use the notation (11) in further narration.

# State estimation

### Measurements

Considering the pipeline system shown in Fig. 1 one assumes a number of pressure sensors installed on the pipe, located quite on a long distance from each other. These sensors measure pressure with known frequency and it is supposed that any systematical error in these measurements is absent. Conversely, the randomized error takes place as a noise.

Consider the time moment $t = N\tau$ as a current moment. The vector $\mathbf{z}_N$ characterizes the current state at this moment and we have sensor measurements from time interval $t \in [0, N\tau]$. This is the data history and it will be used in identification procedure, described below.

The identification problem is stated as follows. One has to find velocity and pressure profiles at $t = 0$, i.e. vector $\mathbf{z}_0$.Then the current state $\mathbf{z}_N$ can be calculated using (11).

## Quasi-linearization

In general, the vector-function $\psi$, executing the transition from layer $n$ to layer $n+1$ is nonlinear.

On the other hand, it is always easier to work with the linear models. We will use the quasi-linearization method [3] to make the transition from $\mathbf{z}_n$ to $\mathbf{z}_{n+1}$ linear:

$$\overset{\mu+1}{\mathbf{z}}_{n+1} = \psi\left(\overset{\mu}{\mathbf{z}}_n\right) + \frac{\partial \psi}{\partial \mathbf{z}}\left(\overset{\mu}{\mathbf{z}}_n\right) \times \left(\overset{\mu+1}{\mathbf{z}}_n - \overset{\mu}{\mathbf{z}}_n\right), \quad (13)$$

$$\forall n = 0,1,\ldots N-1$$

This method is the iterative one. The index μ above the letter represents the iteration number. The second term in the right part of (13) is the Jakobi matrix multiplied by the vector. This matrix is calculated analytically. Every element of this matrix is a partial derivative. The analytical form of this derivative depends on the function ψ, and therefore on the kind of difference scheme chosen. That is as far as the difference scheme is chosen we obtain the formulas for calculation vector-function $\psi$ and Jakobi matrix elements. So long as $\overset{\mu}{\mathbf{z}}_n, \forall n$ at the iteration μ is known one can calculate $\psi\left(\overset{\mu}{\mathbf{z}}_n\right)$ and $\frac{\partial \psi}{\partial \mathbf{z}}\left(\overset{\mu}{\mathbf{z}}_n\right)$ using the described formulas. Thus the equation (13) can be rewritten as:

$$\overset{\mu+1}{\mathbf{z}}_{n+1} = \overset{\mu}{J}_n \times \overset{\mu+1}{\mathbf{z}}_n + \overset{\mu}{\mathbf{w}}_n, \quad \forall n = 0,1,\ldots N-1 \quad (14)$$

where

$$\overset{\mu}{J}_n = \frac{\partial \psi}{\partial \mathbf{z}}\left(\overset{\mu}{\mathbf{z}}_n\right), \quad \overset{\mu}{\mathbf{w}}_n = \psi\left(\overset{\mu}{\mathbf{z}}_n\right) - \overset{\mu}{J}_n \times \overset{\mu}{\mathbf{z}}_n,$$

$$\forall n = 0,1,\ldots N-1.$$

Equation (14) is the linear dependence between $\mathbf{z}_{n+1}$ and $\mathbf{z}_n$ at the new iteration μ+1. Actually, this equation represents the linearized model of the process occurred in the pipeline system. Using this dependence one can derive [3] the following:

$$\overset{\mu+1}{\mathbf{z}}_n = \overset{\mu+1}{\mathbf{Q}}_n \times \overset{\mu+1}{\mathbf{z}}_0 + \overset{\mu+1}{\mathbf{q}}_n, \forall n = 0,1,\ldots N-1, \quad (15)$$

Matrices **Q** and vectors **q** can be explicitly calculated from J and w:

$$\begin{cases} \overset{\mu+1}{\mathbf{Q}}_{n+1} = \overset{\mu+1}{J}_n \overset{\mu+1}{\mathbf{Q}}_n, \forall n = 0,1,\ldots N-1 \\ \overset{\mu+1}{\mathbf{Q}}_0 = \mathbf{I} \end{cases}, \quad (16)$$

$$\begin{cases} \overset{\mu+1}{\mathbf{q}}_{n+1} = \overset{\mu+1}{J}_n \overset{\mu+1}{\mathbf{q}}_n, \forall n = 0,1,\ldots N-1 \\ \overset{\mu+1}{\mathbf{q}}_0 = \mathbf{0} \end{cases}, \quad (17)$$

In the system (16) **I** is the identity matrix.

The equation (15) gives us the dependence of velocity or pressure at any point of the calculation area, at any moment of time from the initial state vector $\mathbf{z}_0$.

## Least Squares Application

As far as we know matrices **Q** and vectors **q**, one can express every measured pressure (see Section 3.1) via initial state vector $\mathbf{z}_0$. If we number all the measurements with some index $i$ (it does not matter what sensor this measurement from), all the measurements can be expressed with vector $\boldsymbol{\varphi}$ where $\boldsymbol{\varphi}_i$ is one of the measurements.

So, if we have $N_{measured}$ measurements from every of $S$ sensors, then $i \in 1,\ldots,S \cdot N_{measured}$ and the dimension of $\varphi$ is $S \cdot N_{measured}$.

The model estimation of $\varphi$ at every iteration $\mu$ is:

$$\hat{\varphi} = \mathbf{A}\mathbf{z}_0 + \mathbf{a} \quad (16)$$

where the rows of the matrix **A** and the vector **a** can be expressed via rows of **Q** and **q** uniquely. It does not matter if the sensor coordinate coincides with a node of the grid, because if not we are always able to do interpolation.

The estimation of $\mathbf{z}_0$ at every iteration μ can be calculated via least squares method, i.e. minimizing the functional:

$$\hat{\mathbf{z}}_0 = \min_{\mathbf{z}_0} \sum_{i=1}^{S \cdot N_{measured}} \left(\varphi_i - \hat{\varphi}_i\right)^2 = \\ = \min_{\mathbf{z}_0} \sum_{i=1}^{S \cdot N_{measured}} \left(\varphi_i - A_i \mathbf{z}_0 - \mathbf{a}_i\right)^2, \quad (17)$$

Here, $A_i, \mathbf{a}_i$ are i-th row of the matrix A and i-th element of the vector **a**. This is ordinary least squares method.

The solution of (17) has the form (Ljung, 1999):

$$\hat{\mathbf{z}}_0 = \frac{\sum_{i=1}^{S \cdot N_{measured}} A_i^T \left(\varphi_i - \mathbf{a}_i\right)}{\sum_{i=1}^{S \cdot N_{measured}} A_i^T A_i}, (18)$$

Thus we have estimated initial state vector at iteration μ. Using this estimation one can calculate the current state vector

$\mathbf{z}_N$ and at the same time calculate all the intermediate state vectors $\overset{\mu}{\mathbf{z}}_n, n = 1,\dots,N-1$. Then, we go to the next iteration μ+1 and use these vectors to calculate $\overset{\mu}{J}_n$ and $\overset{\mu}{\mathbf{w}}_n$, $n = 1,\dots,N-1$.

## NUMERICAL EXPERIMENT

Several tests of the identification method described above have been done. These tests have been performed on the mathematical model and the results of one of it are presented here.

### Modelling parameters

The modeled pipeline has the following parameters: length is 118.061 miles (190 km), inner diameter is 28.346 inches (0.7 m). The elevation has been taken from existing pipeline. The grid parameters: M = 20, N = 10, h = 6.214 miles (h = 10 km), τ = 10 sec. The physical properties of the fluid (density and viscosity) have been taken the same as the typical physical properties of oil: $\rho = 53.69\ \mathrm{lb/ft^3}$, $(860\ \mathrm{kg/m^3})$ $\nu = 10\mathrm{sSt}$.

The modelling of the transient processes has been carried out using (11).

### Initial state

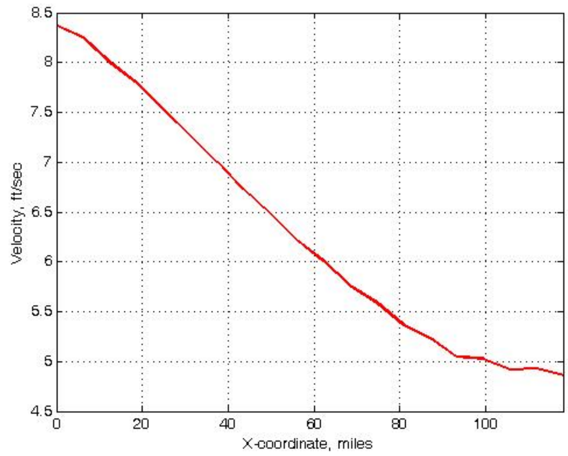
The initial state $\mathbf{z}_0$ has been obtained from the stationary regime by varying the pump number of revolutions. First, there was stationary regime with the next parameters

| Parameter | English unit | SI unit |
|---|---|---|
| $p_{left}$ | 43.5 psi | 0.3 MPa |
| $p_{right}$ | 14.5 psi | 0.1 MPa |
| $u$ | 4.36 ft/sec | 1.33 m/sec |
| $N_r$ | 2000 rev/min | 2000 rev/min |

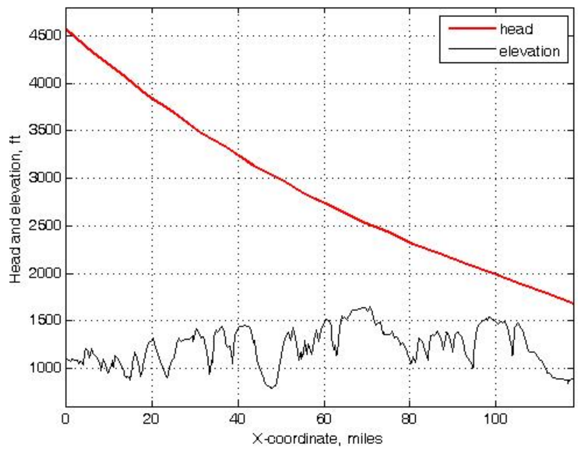
At the time t = -220 sec, $N_r$ has been changed to 2800 rev/min. Then, the revolutions number was maintained constant. At t = 0 velocity and pressure in the pipe had the distributions shown in Fig. 3. The y-axis in Fig. 3b has the dimension of m.

### Sensor measurements imitation

There were considered 7 points with measuring pressure. Coordinates of these points are 0, 18.64, 37.28, 55.92, 74.56, 93.21, 118.06 miles (0 km, 30 km, 60 km, 90 km, 120 km, 150 km, 190 km). Therefore, at every time step n the values $\mathbf{z}^n_{M+1}$, $\mathbf{z}^n_{M+4}$, $\mathbf{z}^n_{M+7}$, $\mathbf{z}^n_{M+10}$, $\mathbf{z}^n_{M+13}$, $\mathbf{z}^n_{M+16}$, $\mathbf{z}^n_{M+20}$ have been saved as a data for identification of $\mathbf{z}_0$. Moreover, these values have been noised with normally distributed noise (noise amplitude = 0.58 psi = 4kPa). It is worth to say that nowadays the measurement accuracy of modern pressure sensors is less or equal 0.29 psi (2kPa). But such noise amplitude has been taken to show the noise effect on the identification procedure.




a.



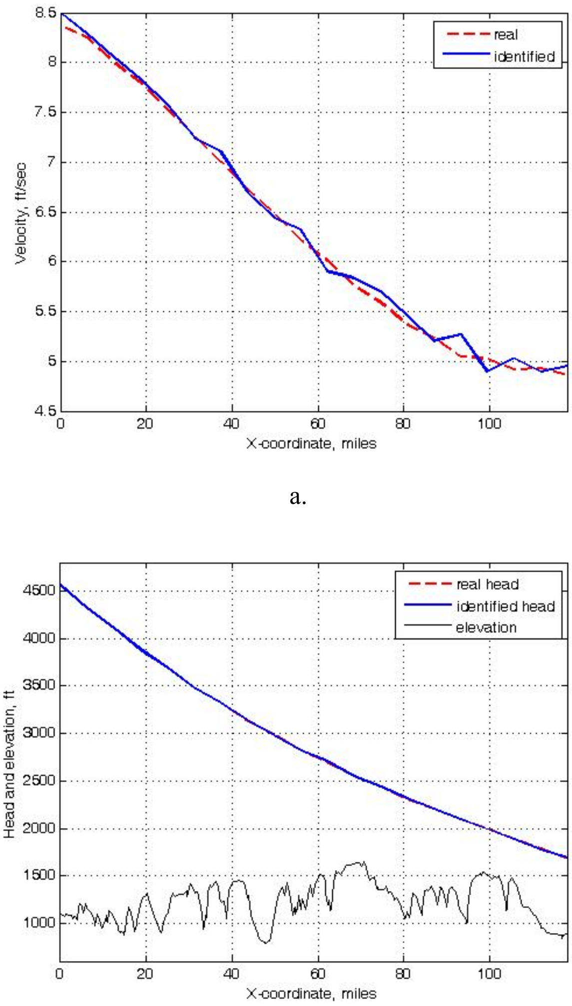

b.

Fig. 3. Initial state of the pipeline

There were considered 10 measurements at every sensor, i.e. matrix $A$ at (16) had a size of $70 \times 40$.

### Results

On the basis of data obtained the identification algorithm has achieved the result as follows. The identification algorithm has converged after 4 iterations, described in Sections 3.2 – 3.3. The identified initial state is depicted in Fig. 4.




a.

b.

Fig.4 Identified initial state of the pipeline.

One can see the divergence between the real initial state and the identified one, looking at the velocity plot (see Fig. 4a). This divergence is due to the noise of measurements. It might seem that there are no any discrepancies in the head plots, but it is not so. The point is that measurement noise is too small and relative error of measurements is less than 0.1%. That is one can see that error in pressure measuring have a slight influence on the results of pressure identification, but entail the perceptible error in velocity definition. It was obtained that the bigger measurements noise entail the bigger error in velocity identification. Considering ideal case without noise we will obtain the exact value of $\mathbf{z}_0$ , i.e. in this case $\hat{\mathbf{z}}_0 = \mathbf{z}_0$ .

As far as initial state has been identified we may model the process occurred. One can take the obtained estimation $\hat{\mathbf{z}}_0$ as the initial state and calculate $\hat{\mathbf{z}}_n, n = 1,\ldots N$ , via (11).

## CONCLUSIONS

The problem of current identification in the pipeline system has been considered in the view of linearized model. The model equations stated in the Section 2 have been linearized using the quasi-linearization method and the iteration algorithm of past initial state identifying has been constructed.

The results of numerical experiment using this algorithm are given. They show the influence of the measurements noise on the identification precision.

The identification method described in this paper works with the process modelled.

The ideas stated in the paper can be easily extended to other elements of modern pipelines such as T-joint, regulators, filters, etc. Therefore, these ideas give a powerful instrument for state estimation in complex pipeline systems. Thus these ideas are supposed to find practical application.

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