

PSIG 1002

Two-Dimensional Effects in Pipe Flow

Jerry L. Modisette, Ph.D., Consultant, Jason Modisette, Ph.D., ATMOS International

Copyright 2010, Pipeline Simulation Interest Group

This paper was prepared for presentation at the PSIG Annual Meeting held in Bonita Springs, Florida, 11 May – 14 May 2010.

This paper was selected for presentation by the PSIG Board of Directors following review of information contained in an abstract submitted by the author(s). The material, as presented, does not necessarily reflect any position of the Pipeline Simulation Interest Group, its officers, or members. Papers presented at PSIG meetings are subject to publication review by Editorial Committees of the Pipeline Simulation Interest Group. Electronic reproduction, distribution, or storage of any part of this paper for commercial purposes without the written consent of PSIG is prohibited. Permission to reproduce in print is restricted to an abstract of not more than 300 words; illustrations may not be copied. The abstract must contain conspicuous acknowledgment of where and by whom the paper was presented. Write Librarian, Pipeline Simulation Interest Group, P.O. Box 22625, Houston, TX 77227, U.S.A., fax 01-713-586-5955.

ABSTRACT

While single-phase pipe flow is almost always assumed to be one-dimensional, that is, homogeneous across the pipe's diameter, there can potentially be two-dimensional thermal effects in the laminar flow of high-viscosity oil. The heat that's initially imparted to the oil by heaters or by the pumping inefficiencies must conduct through the outer part of the oil before it can leave the pipe. If the oil has a strongly temperature-dependent viscosity, this can lead to significant viscosity variations across the pipe, and in turn to a flow regime that differs significantly from the usual Hagen-Poiseuille solution for laminar pipe flow.

This article investigates the conditions necessary for the one-dimensional approximation to introduce significant errors in the frictional pressure gradient.

INTRODUCTION

Flow in pipelines is generally assumed to be one-dimensional. That is, the pressure, temperature, and velocity are assumed to be uniform across the pipe. The one-dimensional approximation for the pressure is quite accurate under nearly all conditions, since any pressure gradient across the pipe will cause the fluid to move so as to balance the pressure. There is a small increase in pressure from the top of the pipe to the bottom to balance the gravitational head, and from the inside to the outside of a bend to balance the centrifugal force of the flow turning around the bend. For pipe lines these variations are small fractions of the pressure in the pipe. Actually, it is the head that is constant across the pipe.

There are variations in the velocity and the temperature near the pipe wall, because the temperature of the environment is generally different from that of the fluid and the velocity of the wall is, of course, zero. For turbulent flow the eddy conductivity is so high that the constant temperature approximation is still valid, although not perfectly accurate. Heat transfer to the pipe wall can be calculated by a heat transfer coefficient or, usually, by calculating the conduction in the pipe wall and the surrounding environment.

There is always a boundary layer near the pipe wall in which the velocity varies from the bulk fluid velocity to zero at the wall. For turbulent flow this boundary layer is very small compared to the pipe radius. The momentum transfer, which we usually refer to as the frictional resistance to the flow, can be calculated from the Darcy-Weisbach equation with a friction factor which depends on pipe roughness and the Reynolds number. For laminar flow, the boundary layer essentially fills the pipe. That is, the velocity varies all across the pipe. Steady-state isothermal laminar flow in a pipe, also known as Hagen-Poiseuille flow is one of the few fluid flow problems for which an exact solution can be found¹. The velocity profile across the pipe is a parabola, with zero velocity at the wall and a centerline velocity equal to twice the

¹ Other exact solutions include the Bernoulli equation describing flow of an incompressible fluid in a tube with variable diameter and elevation, isentropic flow in a nozzle, Poiseuille flow between two flat plates moving in opposite directions, Prandtl-Meyer flow around a corner, and the Rankine-Hugoniot relations describing the changes in velocity, temperature, and pressure across a shock wave. There are also a few from astrophysics including Sweets mechanism for the merging of two plasmas containing oppositely directed magnetic fields, which is the energy source for solar flares, Parkers solution of the solar wind equations for isothermal flow, and one of the authors (JLM) calculation of the torque on the sun caused by the solar wind dragging out magnetic fields. All of these solutions involve simplifying assumptions to make the problem solvable. Most real fluid-flow problems require numerical solution techniques.

average velocity. From this solution an exact friction factor for the Darcy-Weisbach equation can be calculated.

This paper addresses situations in which the uniform flow approximation is not valid, making it necessary to consider two-dimensional effects.

Significant two-dimensional effects arise when there is a large temperature difference between the fluid and the environment and the fluid viscosity is strongly dependent on temperature, as in the case of heavy petroleum. The large temperature difference can make the rate of heat loss so great that there is a significant radial temperature gradient in the fluid. The temperature gradient produces a higher viscosity near the pipe wall, lowering the velocity. For turbulent flow, the eddy conductivity is large enough that the temperature gradient is insignificant, even with large temperature differences.

Turbulent mixing is a very efficient heat transfer mechanism. However, in laminar flow, all heat transfer must occur through conduction, which can be slow enough that temperature gradients can persist, resulting in a radial velocity distribution that is quite different from the usual 1D constant-viscosity form.

If the center of a pipe in laminar flow were to become warm enough, the viscosity might drop to the point where some part of it went into turbulent flow. When calculating the Reynolds Number for a central cylinder of oil of smaller diameter than the pipe, the smaller diameter and smaller relative velocity would suggest a lower Reynolds Number, but since the viscosity is usually an exponential function of temperature, that suggests that the temperature differences might overwhelm the velocity and diameter differences and lead to turbulence.

LAMINAR FLOW IN A PIPE

Nineteenth century experiments by Joshua Reynolds showed that laminar flow occurs at Reynolds numbers below 2100. The Reynolds number is calculated from:

$$\text{Re} = \frac{\rho v D}{\mu} \quad (1)$$

where ρ is the density of the fluid measured in lbs/cu ft, v is the velocity in ft/sec, D is the pipe diameter in feet, and μ is the dynamic viscosity in lbs/sq ft/sec. The Reynolds number is usually interpreted as a measure of the ratio of the inertial forces to the viscous forces.

For Reynolds numbers above 2100, the flow can be laminar, but it is unstable, tending more to turbulence as the Reynolds number increases. Friction factor curves usually refer to regions near the laminar flow line as partially turbulent. This reference does not have a rigorous physical basis, but conforms to the idea that the eddies (vortices) constituting turbulence form slower and decay faster at lower Reynolds numbers.

VELOCITY DISTRIBUTION IN LAMINAR FLOW

This derivation can be found in fluid dynamics textbooks, if one is sufficiently determined; however, it was not included in

the texts we had on hand. The authors include it partly for the convenience of the reader, partly so we will know where to find it in future and partly to show we can still do it.

If we consider a thin cylindrical shell of fluid inside a pipe, with the cylinder concentric with the pipe, in steady flow the net viscous force on the shell must balance the pressure gradient force. This requirement is described by:

$$\begin{aligned} \frac{d}{dr} 2\pi\mu r \frac{dv}{dr} &= \frac{dP}{dx} \frac{d}{dr} \pi r^2 \\ &= -2\pi r \frac{dP}{dx} \end{aligned} \quad (2)$$

where r is the radius of the shell in feet, P is the pressure in lbs/sq ft, and x is the distance along the pipe in feet.

Dividing both sides of (2) by 2π and integrating from the center of the pipe to r :

$$\mu r \frac{dv}{dr} = \frac{r^2}{2} \frac{dP}{dx} \quad (3)$$

in which we have invoked the fact that at the center of the pipe, $dv/dr = 0$ by symmetry. Integrating again gives

$$\mu(v(r) - v(0)) = \frac{r^2}{4} \frac{dP}{dx} \quad (4)$$

which leads us to the classical laminar flow parabolic velocity distribution of Hagen-Poiseuille flow:

$$v = v(0) + \frac{dP}{dx} \frac{r^2}{4\mu} \quad (5)$$

Note that the pressure gradient is negative, so the velocity is at a maximum in the center of the pipe and drops off towards the walls. In fact at the pipe wall the velocity must be zero, so (5) can be solved for $v(0)$,

$$v(0) = -\frac{dP}{dx} \frac{R^2}{4\mu} \quad (6)$$

and so the complete velocity profile is

$$v(r) = \frac{dP}{dx} \frac{(r^2 - R^2)}{4\mu} \quad (7)$$

RAMIFICATIONS OF THE LAMINAR VELOCITY DISTRIBUTION

The average velocity in laminar flow can be calculated by integrating the velocity from equation (7) weighted by the cross-sectional area $2\pi r dr$ of a shell of thickness dr at radius r from the center of the pipe, and then dividing the result by the cross-sectional area of the pipe πR^2 :

$$\begin{aligned}
 v_{avg} &= \frac{1}{4\mu} \frac{dP}{dx} \int_0^R 2\pi r dr (r^2 - R^2) \\
 &= -\frac{R^2}{8\mu} \frac{dP}{dx} \\
 &= \frac{v(0)}{2}
 \end{aligned} \tag{8}$$

again, a well-known classical result.

Now consider a cylinder of fluid, not a shell, concentric with the pipe of radius r . This cylinder of fluid may be thought of as a pipe of radius r with the wall moving at velocity $v = v(r)$. Therefore the relevant velocity for determining the Reynolds number for flow in the cylinder is the difference between the average velocity in the cylinder and the outer surface velocity v .

The average velocity (relative to the pipe wall) is given by equation (8) with $R = r$:

$$v_{avg} = v(0) + \frac{r'^2}{8\mu} \frac{dP}{dx} \tag{10}$$

Subtracting v' to get the average velocity relative to the cylinder wall:

$$\begin{aligned}
 v'_{avg} &= v(0) + \frac{r'^2}{8\mu} \frac{dP}{dx} - v(0) - \frac{r'^2}{4\mu} \frac{dP}{dx} \\
 &= \frac{r'^2}{8\mu} \frac{dP}{dx}
 \end{aligned} \tag{11}$$

Now the velocity at the center of the pipe relative to the cylinder wall is

$$\begin{aligned}
 v'(0) &= v(0) - v(0) - \frac{r'^2}{4\mu} \frac{dP}{dx} \\
 &= -\frac{r'^2}{4\mu} \frac{dP}{dx}
 \end{aligned} \tag{12}$$

So the relative velocity at the center of the cylinder is again twice the average relative velocity. (This is a general property of a parabola).

Now let us consider the Reynolds number in the relative system. From equation (11) the average relative velocity increases with the square of the cylinder diameter. The Reynolds number increases with the diameter and with the average velocity. Therefore, the Reynolds number based on the relative velocity of the flow in any concentric cylinder is always less than the Reynolds number for the overall pipe flow. Laminar flow in the pipe as a whole always produces conditions for laminar flow in the inner cylinders.

These statements are only true for constant viscosity. We will be considering cases where the viscosity is higher in the outer

cylinders, so that we may have turbulent flow in the inner cylinders even though the laminar flow criterion is satisfied in the outer layers.

VELOCITY DISTRIBUTION IN TURBULENT FLOW

Turbulent flow is commonly described as having many small, random motions of fluid elements. A fluid element is a small region of fluid that moves in approximately the same direction. However, the motion of fluid elements cannot quite be random, because two fluid elements cannot occupy the same space at the same time and fluid cannot move so as to leave an empty hole.

The space into which a fluid element moves must be vacated by the fluid already there. The motion of adjacent fluid elements must be coordinated, although they may be so complex as to appear random. In laminar flow the motion is coordinated by the fluid elements along a streamline moving in the same direction at the same velocity. Changes in velocity (in a pipe of constant cross-section) are accompanied by changes in the density so that fluid elements shrink or expand to make room or occupy space.

A vortex allows fluid elements to move on circular streamlines, with each fluid element making room for the element behind it in the circle. There can be other, less organized paths, but the motion of adjacent elements must be coordinated so that room is made for moving elements and so that no holes are made in the fluid.

Reynolds original experiments on turbulence in pipes were made using transparent tubes and hypodermic needles injecting a small stream of ink in the flow. The effect of vortices could be seen in the ink trails, which typically followed cycloids or epicycles, which are the paths of points on a translating, spinning disk.

The driving force that causes turbulence is an instability called the twostream instability. Two adjacent streamlines at different velocities are unstable. A small deviation in the direction of a fluid element pushes into the adjacent streamline. The inertia of the fluid in the streamline resists the deviating element, pushing it back. Unless the viscosity is enough to damp the motion, the streamline oscillates at progressively larger amplitude until the oscillating elements produce a vortex. This effect can be seen in ocean waves, which are manifestations of the two-stream instability. For ocean waves, the oscillation is damped by gravity. When a wave approaches a sloping beach, the gravitational damping decreases because the depth of the water decreases, decreasing the gravitational head. Ultimately, the wave breaks, sometimes producing a moving vortex which surfers call the curl.

Another example of the two-stream instability producing a vortex is a canoe paddle. If one digs the paddle in and pulls hard, vortices can be seen shedding from the paddle as it moves through the water. This is inefficient paddling because the energy in the spinning vortex, which came from the

paddlers work, does not help move the canoe forward.

In a pipe making a transition from laminar to turbulent flow, vortices are generated at the wall, because the velocity changes faster at the wall; the parabola is steeper there. Once established, a vortex has a life of its own. It can migrate through the fluid, still subject to the restriction that room must be made for it by the motion of other vortices or fluid elements not part of vortices. In turbulent flow, vortices are continually generated at the wall, moving out into the flow stream. Vortices are continually decaying due to viscous forces that resist the relative motion. Two adjacent vortices decay even faster because their adjacent edges are moving in opposite directions. However, two vortices pushed strongly together can merge, creating one larger vortex.

Readers may think we have beaten the vortex concept to death, but it is important to understand what is physically happening in turbulent flow. For flow in pipelines, we are generally more interested in the gross effects of turbulence than in the microscopic details. For this reason, we describe the velocity profile in terms of the local average velocity. The velocity is either averaged over a distance scale large compared to a turbulent eddy but generally small compared to the pipe diameter or it is averaged over a time large compared to the circulation time in an eddy. (Both of these averages turn out to be the same.) The point is that circulation in an eddy or vortex does not move fluid down the pipe. For either of the averages just described, the circulation cancels out; that is, the component of the circulation along the pipe averages out to zero. Only the motion of the vortex itself contributes to the net movement of fluid down the pipe.

We also calculate the frictional resistance in terms of average velocities. The details of the production, decay, and interaction of vortices are buried in the friction factor.

In turbulent flow, the velocity distribution, in terms of the local average velocities just described, is almost constant across the pipe, quickly falling to zero in the thin laminar boundary layer. The very high velocity gradient in the boundary layer produces a large viscous drag from the pipe wall, which is why turbulent friction factors are much larger than laminar friction factors.

EFFECT OF VARYING THE VISCOSITY ACROSS THE PIPE

The viscosity of fluids varies with temperature. For gases, the viscosity increases with temperature because the molecules move faster, thus transferring momentum faster across a velocity gradient. For most liquids, the viscosity decreases with temperature, because at higher temperature the molecules are not as close together, resulting in less friction between adjacent molecules.

A notable exception is sulfur. At low temperatures, the molecules in melted sulfur are eight-atom rings. As the temperature increases, the rings break, becoming linear chains

that interfere with one another. The sulfur goes from having the consistency of water to being like syrup. This example is provided as a warning to be careful about generalities.

When oil first enters a pipe after coming out of a pump station, it will be pretty well-mixed, having just passed through pumps, bends, and various other equipment. We can therefore assume that the temperature is constant across the pipe at that point. If the pipe is in laminar flow, this means the same layer of oil will be flowing right next to the wall for its entire length. This oil will initially have a very large temperature difference from the pipe wall, and so will lose heat rapidly through conduction.

Eventually the oil in the pipe will reach a steady state where the cross-sectional temperature profile is what's necessary to conduct the heat generated by viscous dissipation out of the pipe. However, this can be a very long "eventually", especially for large pipes.

In between the initial constant-temperature state and the final steady state, the oil will have a hotter-than-steady-state core as some of the initial heat will have not conducted out of the pipe yet. We expect the oil to cool more slowly in this case than it would if perfectly mixed: in the perfectly mixed case, the temperature difference between the hot core of the oil and some outer ambient temperature would occur over a longer distance than it would in the laminar-flow case, as it would start at the edge of the pipe rather than the center of the pipe. This means the temperature gradient would be larger (due to having the same delta temperature over a shorter distance) and therefore the rate of heat flow due to conduction would be larger as well.

SIMULATION RESULTS

Two simulators were developed for this work, one that explicitly assumes no change in fluid properties (except for pressure) along the pipe, that is, that the fluid has reached its final steady state, and one that simulates the whole pipe from the inlet on. Both simulators assume the oil has reached a steady state in time, i.e. that there are no time changes of temperature, velocity, or pressure.

Direct Simulation of the Final State

In the final state we know that the radial velocity must be zero, since any other value would involve perpetual sourcing or sinking of oil. This can be seen from the conservation of mass equation in steady state in the incompressible limit (which can be found in e.g. the incompressible Navier- Stokes equations):

$$\nabla \cdot \vec{v} = \frac{dv_r}{dr} + \frac{dv_x}{dx} = 0 \quad (13)$$

where v_r is the radial velocity, v_x is the axial velocity (the velocity along the pipe), r is the radial coordinate and x is the axial coordinate.

Since in the final state all derivatives with respect to x are zero, dv_r / dr must also be zero, which means v_r must be a constant and since we know v_r is zero at the pipe wall, v_r must

be zero everywhere.

The only two simulation variables in the final-state simulation are then axial velocity v_x and temperature T . The pressure gradient is assumed to be constant, and the fluid is treated as incompressible (since the density changes due to the small pressure and temperature changes across the diameter of the pipe are negligible.)

The two equations governing this are the conservation of axial momentum and the conservation of energy. The energy equation includes a term for viscous dissipation and for conduction, which must cancel out since the usual convection terms are zero in the final-steady-state limit. Radial momentum is conserved: it's zero everywhere, as stated above. And if the radial velocity is zero, and the fluid is incompressible, and the axial velocity doesn't change along the pipe, then mass is conserved as well.

The axial momentum equation solved for this model was

$$-\frac{dP}{dx} + \mu \left(\frac{1}{r} \frac{dv_x}{dr} + \frac{d^2v_x}{dr^2} \right) + \frac{d\mu}{dr} \frac{dv_x}{dr} = 0 \quad (14)$$

and the energy equation was

$$k \left(\frac{d^2T}{dr^2} + \frac{1}{r} \frac{dT}{dr} \right) + \mu \left(\frac{dv_x}{dr} \right)^2 = 0 \quad (15)$$

where k is the thermal conductivity of the oil.

These two nonlinear partial differential equations were solved in cylindrical coordinates, with angular terms all assumed to be zero, that is, assuming cylindrical symmetry. At the wall, the axial velocity is zero and the heat flux must match the heat flux through the wall, which can be calculated using a heat loss coefficient for the wall and the oil temperature at the wall. At the center of the pipe, the derivatives with respect to radius of the axial velocity and the temperature are set to zero. This form of the central boundary condition is necessary, because otherwise there would be a cusp in the pipe properties at the center, which would be non-physical.

These equations were coupled to a simple equation for temperature dependence of dynamic viscosity of the form

$$\mu(T) = A \exp\left(\frac{B}{T}\right) \quad (16)$$

where A and B are constants determined by fitting two viscosity points.

Final-State Model Results

We ran this model for a variety of different systems: pipes of 10", 20", 30", and 40" inside diameter with ambient temperatures of 0, 20, and 40 deg F and fluid kinematic viscosities of either 5 cSt @ 60 deg F and 10 cSt @ 40 deg F, four times this viscosity, or ten times this viscosity. The pipe was clad in a 3"-thick layer of concrete, the outside of which was assumed to be at the ambient temperature.

The calculated flow (either at a fixed pressure gradient of 5 psi/mile, or a fixed Reynolds Number of 4000, whichever was

more restrictive of flow) was compared with the flow calculated for the same system using a traditional Hagen-Poiseuille solution where the fluid was assumed to be perfectly mixed, that is, the temperature and therefore viscosity are constant all across the fluid. These two conditions ensured that all of these results are at least potentially for laminar flow (this model only applies to laminar systems; a Reynolds Number of 4000 is at the upper end of what might be laminar) and that the pressure gradients were not unreasonable for a real-world pipeline.

The results of these calculations are shown in Table 1. As expected, the calculated flow rate at a given pressure gradient or a given Reynolds Number was always higher for the full 2D model than it was for the case assuming perfect mixing. This happens because the core of the oil is actually better at retaining the heat of viscous dissipation that the perfect-mixing approximation would have it, so it remains at a lower viscosity and has less resistance to flow.

The magnitude of the flow rate difference between the 2D model and the 1D perfect-mixing model was strongly dependent on the scenario. It was significant only for the highest-viscosity cases, and was more important

for larger pipes. Note that all of the cases presented here have pretty high viscosities: at an ambient temperature of 40 degrees Fahrenheit, the viscosity at ambient temperature is twice what's listed in Table 1; at 20 degrees it's 4x as much, and at 0 degrees it's 8x as much. The actual viscosities in the oil will be a bit lower than these multipliers because the oil will be a bit warmer than ambient.

For the 10" and 20" pipes, no effect of greater than a few percent of flow was seen for any set of conditions. However, for the 30" and 40" pipes, flow rate errors were more than 10% for the high-viscosity and low-ambient temperature cases. These flow differences were accompanied by significant temperature variation across the fluid in the modeled case, with the centerline temperature being 5 degrees Fahrenheit greater than the perfectly-mixed case temperature in both of the scenarios with 10%+ differences in flow between the new 2D and traditional 1D models.

Simulation of the Whole Pipe

How long does it take the pipe to reach the final states simulated in the previous section? The answer, it turns out, it "quite long". The authors developed a full 2D simulator (2D and not 3D because it assumes radial symmetry) of mass, momentum, and heat transfer in a pipe. The simulator simultaneously solves the four coupled conservation equations for mass, radial momentum, axial momentum, and energy in steady state. In this simulation the oil was no longer treated as incompressible, but as having a constant bulk modulus K and thermal expansion coefficient β , so that the density could be written

$$\rho(P, T) = \rho(P_0, T_0) \exp\left(\frac{P - P_0}{K} - \beta(T - T_0)\right) \quad (17)$$

The purpose of this simulation was to see how long the oil took to reach the final state found above in some real-world situations and what its behavior was like before it got there. The whole-pipe model was validated against the results of the final-state model, and found to agree.

The boundary conditions used for the whole-pipe model were that the wall be at the ambient temperature (so, no concrete cladding in this case- the authors were looking for a minimum estimate of the time required to reach the final state in the case with the highest heat loss). The oil at the upstream end of the system was assumed to be well-mixed and so therefore at a constant temperature. The radial velocity was set to zero at the upstream end, the wall, and the center of the pipe. The axial velocity was set to zero at the wall and forced to have a maximum at the center of the pipe for symmetry, and to follow the Hagen-Poiseuille profile 7 at the upstream end. The pressure was set to a constant value at the upstream end, and forced to have $dP/dr = 0$ at the pipe center (due to symmetry) and at the pipe wall.

This model was then solved in steady state for the pressure, axial and radial velocities, and temperature, use a method of successive linearizations of the discretized nonlinear conservation laws.

The four balance equations used are

$$v_r \left(\frac{1}{K} \frac{dP}{dr} - \beta \frac{dT}{dr} \right) + v_x \left(\frac{1}{K} \frac{dP}{dx} - \beta \frac{dT}{dx} \right) + \frac{dv_r}{dr} + \frac{dv_x}{dx} = 0$$

$$-\rho \left(v_x \frac{dv_r}{dx} + v_r \frac{dv_r}{dr} \right) - \frac{dP}{dr} - (\nabla \cdot \tau)_r = 0$$

$$-\rho \left(v_x \frac{dv_x}{dx} + v_r \frac{dv_x}{dr} \right) - \frac{dP}{dx} - (\nabla \cdot \tau)_x = 0 \quad (20)$$

$$-c_p \rho \left(v_x \frac{dT}{dx} + v_r \frac{dT}{dr} \right) + k \nabla^2 T - \tau \cdot \nabla \vec{v} = 0 \quad (21)$$

Here τ is the viscous stress tensor; the parts used in these balance equations are (given radial symmetry):

$$\begin{aligned} -(\nabla \cdot \tau)_r &= \mu \left(\frac{d^2 v_r}{dr^2} + \frac{1}{r} \frac{dv_r}{dr} + \frac{d^2 v_r}{dx^2} \right) \\ &+ \frac{\mu}{3} \left(\frac{1}{r} \frac{dv_r}{dr} - \frac{v_r}{r^2} + \frac{d^2 v_r}{dr^2} + \frac{d^2 v_x}{dx dr} \right) \\ &+ 2 \frac{d\mu}{dr} \frac{dv_r}{dr} + \frac{d\mu}{dx} \left(\frac{dv_r}{dx} + \frac{dv_x}{dr} \right) \\ &- \frac{2}{3} \frac{d\mu}{dr} \left(\frac{v_r}{r} + \frac{dv_r}{dr} + \frac{dv_x}{dx} \right) \end{aligned} \quad (22)$$

$$\begin{aligned} -(\nabla \cdot \tau)_x &= \mu \left(\frac{d^2 v_x}{dr^2} + \frac{1}{r} \frac{dv_x}{dr} + \frac{d^2 v_x}{dx^2} \right) \\ &+ \frac{\mu}{3} \left(\frac{1}{r} \frac{dv_r}{dx} + \frac{d^2 v_r}{dr dx} + \frac{d^2 v_x}{dx^2} \right) \\ &+ \frac{d\mu}{dr} \left(\frac{dv_x}{dr} + \frac{dv_r}{dx} \right) + 2 \frac{d\mu}{dx} \frac{dv_x}{dx} \\ &- \frac{2}{3} \frac{d\mu}{dx} \left(\frac{v_r}{r} + \frac{dv_r}{dr} + \frac{dv_x}{dx} \right) \\ -\tau \cdot \nabla \vec{v} &= \mu \left(\left(\frac{dv_x}{dr} + \frac{dv_r}{dx} \right)^2 + 2 \left(\left(\frac{dv_r}{dr} \right)^2 + \left(\frac{v_r}{r} \right)^2 + \left(\frac{dv_x}{dx} \right)^2 \right) \right) \\ &- \frac{2\mu}{3} \left(\frac{dv_x}{dx} + \frac{dv_r}{dr} + \frac{v_r}{r} \right)^2 \end{aligned} \quad (23)$$

In the first (conservation of mass) equation, the explicit form of the equation of state (17) is used.

Whole-Pipe Model Results

The whole-pipe model was more complete than the final-state model described above, in that it actually solved for radial velocity and pressure instead of assuming forms for them. It turned out that in all cases investigated, the radial velocity was close to zero, always less than $1E-5$ ft/sec, and the radial pressure variation was also very small. Results are presented here for a single case, a 32" inside diameter pipe with oil of 0.9 specific gravity, no insulation (the pipe wall held at the ambient temperature of -10 degrees F), oil entering the line at 50 degrees and simulated over 100 miles. The flow is laminar everywhere; the flow rate is 5300 bph. The oil kinematic viscosity ramps from 100 cSt @ 60 degrees to 150 cSt @ 45 degrees and uses an Andrade-type correlation (Equation (16)).

In order to prevent the discontinuous boundary conditions at the upstream end pipe wall from interfering with model convergence, the pipe wall boundary temperature was ramped linearly from the upstream temperature to the ambient temperature over the first 5 miles.

Figure 1 shows the cross-sectional temperature profile at various distances along the pipe. Even after 100 miles, the core temperature is still far above the ambient temperature. In the no-insulation case, a model with perfect mixing such as that used for comparison in the final-state model results section would have the whole body of fluid eventually reach the ground temperature.

Figure 2 shows the average temperature as function of distance for the present model and a 1D model assuming perfect mixing. The 1D model was given 3" of concrete cladding for insulation (otherwise the result would be that it immediately dropped to ground temperature) for comparison of the effectiveness of this sort of insulation vs. the insulating

properties of the oil itself. The 2D result shows a much slower relaxation of average oil temperature to ambient temperature, indicating that the oil is acting effectively as insulation.

The axial velocity does not follow the normal Hagen-Poiseuille parabolic form once the hot core has been established, as can be seen in Figure 3. The cold, high-viscosity oil near the pipe wall is moving more slowly as one would expect. Figure 4 illustrates how the viscosity changes along the pipe for three radial positions: the center (in red) where there is no significant change for the first 50 miles, the edge (which immediately shoots up to the ambient temperature value in the 5-mile initial-ramp region described above), and a point halfway from the center to the edge. The radial distributions of viscosity are illustrated in Figure 5 for various distances along the line. The striking thing about these whole-pipe simulation results is how slowly the fluid approaches the final equilibrium state. It appears that for this 32" pipe it would take several hundred miles for the oil to reach approximately ground temperature. This effect is more prevalent in the cases of very high viscosities as seen here, and will only occur during laminar flow.

CONCLUSIONS

The simulations described above never found the spontaneous formation of a turbulent core. This is not surprising for the initially laminar cases, since, even though the oil retains its

heat much longer than a 1D model would suggest, it still only gets colder and more viscous as it goes down the line, so the Reynolds Number of central regions will just decrease. The only way initially laminar flow could develop a turbulent core would be if in the steady state there was so much heat generation in the central region due to viscous dissipation that the central viscosity dropped enough to allow turbulence. The viscous heat dissipation will be large only if the pressure gradient is large; however, the pressure gradient is only large if the oil is moving pretty fast, which for large pipes means that the flow is already turbulent to start with. The possible central turbulent region is thus limited to case of extremely high-viscosity oil, probably higher than would be expected in real pipelines. For large pipes in laminar flow with very high viscosity oil, the 1D "perfect mixing" approximation that is implicitly used in most pipeline models today can produce significant deviations from the true behavior of the line even in the final equilibrium state far downstream from pumps or heaters, and can produce much larger deviations from reality while reaching that state - a process that might take the entire length of the line between stations.

ACKNOWLEDGEMENTS

Jason Modisette would like to acknowledge the support of ATMOS International in this work.

TABLES

Inner Diameter (inches)	Viscosity @ 60 F (cSt)	Ambient Temperature (deg F)	dP/dx (psi/mile)	Re	Flow (bph)	Flow Error (%)	Temperature Error (deg F)
10	5	40	0.17	4000	183	0	0
10	5	20	0.72	4000	387	0	0.02
10	5	0	3.59	4000	865	0.4	0.24
10	20	40	2.54	4000	727	0.2	0.14
10	20	20	5	1781	681	0.4	0.26
10	20	0	5	343	299	0.2	0.12
10	50	40	5	1272	575	0.3	0.23
10	50	20	5	280	271	0.1	0.1
10	50	0	5	54	119	0	0.05
20	5	40	0.02	4000	366	0	0
20	5	20	0.09	4000	775	0	0
20	5	0	0.46	4000	1750	0	0.06
20	20	40	0.32	4000	1463	0	0.04
20	20	20	1.41	4000	3060	0.6	0.34
20	20	0	5	3203	5133	4.5	2.10
20	50	40	1.93	4000	3586	0.9	0.55
20	50	20	5	2543	4580	3.6	1.86
20	50	0	5	462	1957	1.6	0.78
30	5	40	0.01	4000	546	0	0
30	5	20	0.03	4000	1164	0	0
30	5	0	0.14	4000	2630	0	0.04
30	20	40	0.10	4000	2196	0	0.02
30	20	20	0.42	4000	5629	0.2	0.15
30	20	0	1.96	4000	9914	3.4	1.57
30	50	40	0.59	4000	5447	0.4	0.25
30	50	20	2.34	4000	10826	4.1	2.06
30	50	0	5	2013	11133	13.4	4.73
40	5	40	0.001	4000	732	0	0
40	5	20	0.01	4000	1552	0	0
40	5	0	0.06	4000	3508	0	0.02
40	20	40	0.04	4000	2928	0	0.01

40	20	20	0.18	4000	2928	0	0.01
40	20	0	0.87	4000	6189	0.1	0.08
40	50	40	0.25	4000	7291	0.2	0.15
40	50	20	1.05	4000	14889	2.4	1.27
40	50	0	1.12	4000	21295	12.3	5.46

Table 1 – Steady, final-state effects of viscosity variation across the pipe diameter due to conduction for various pipe sizes and oil viscosities

FIGURES

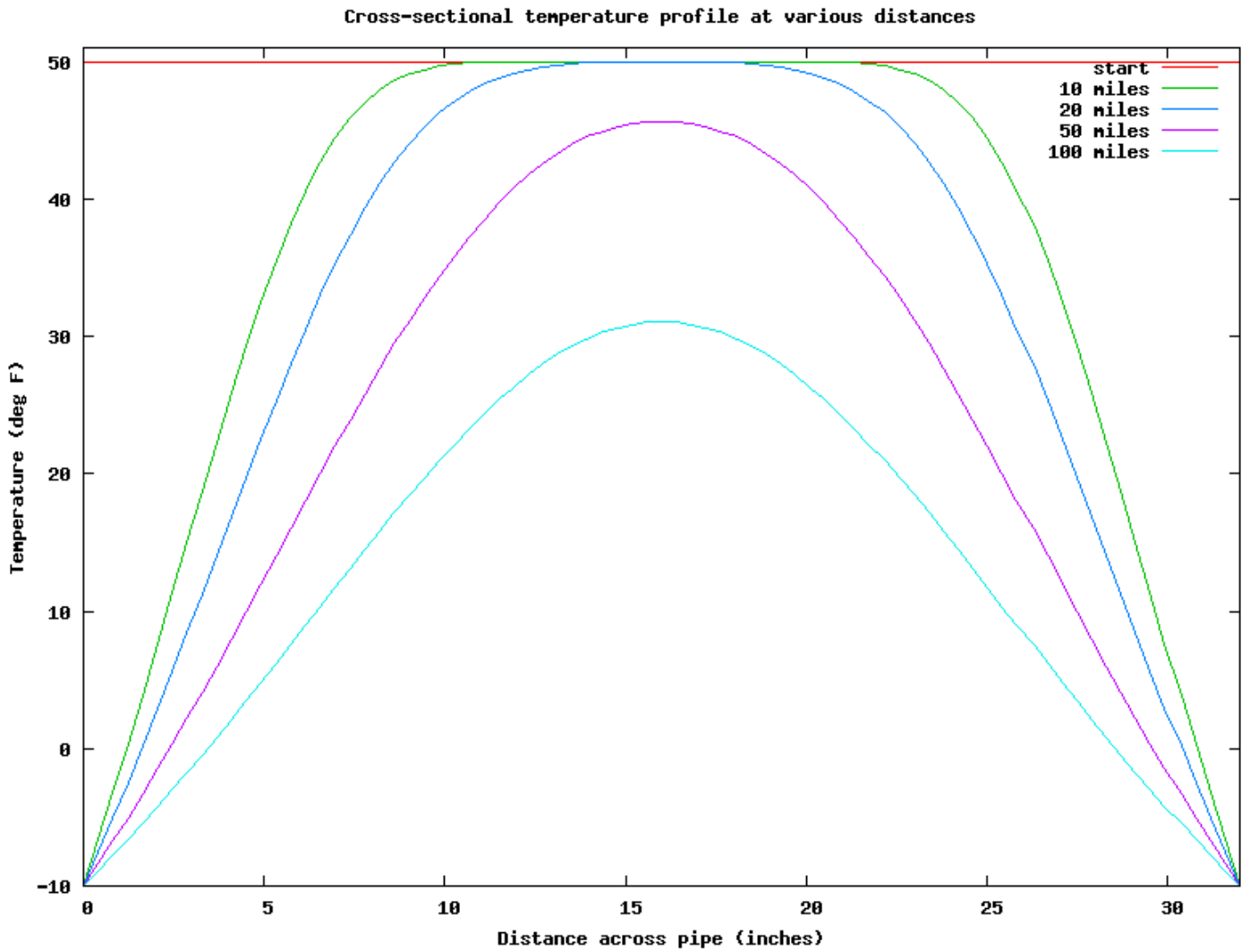


Figure 1 – Temperature across the pipe

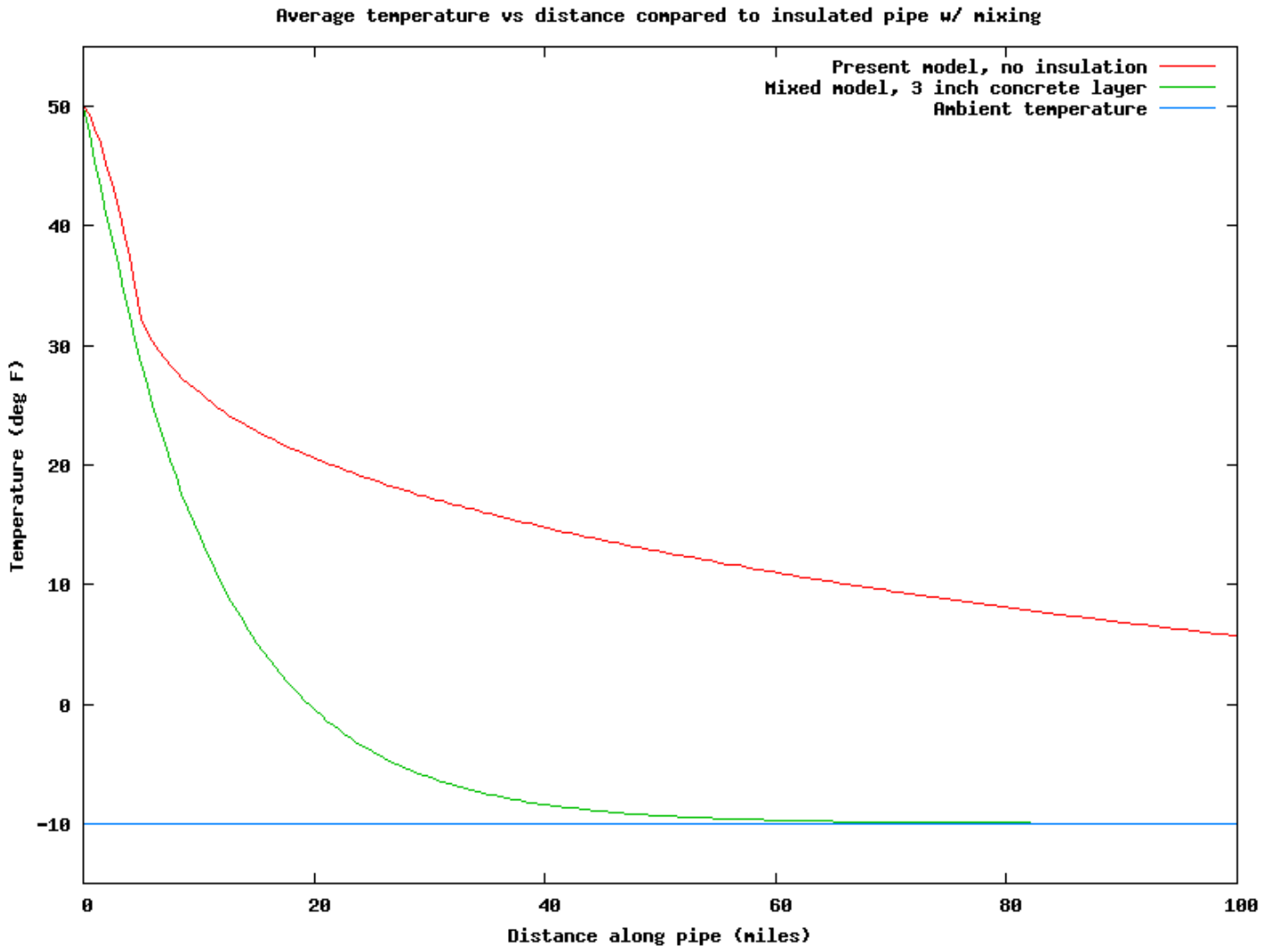


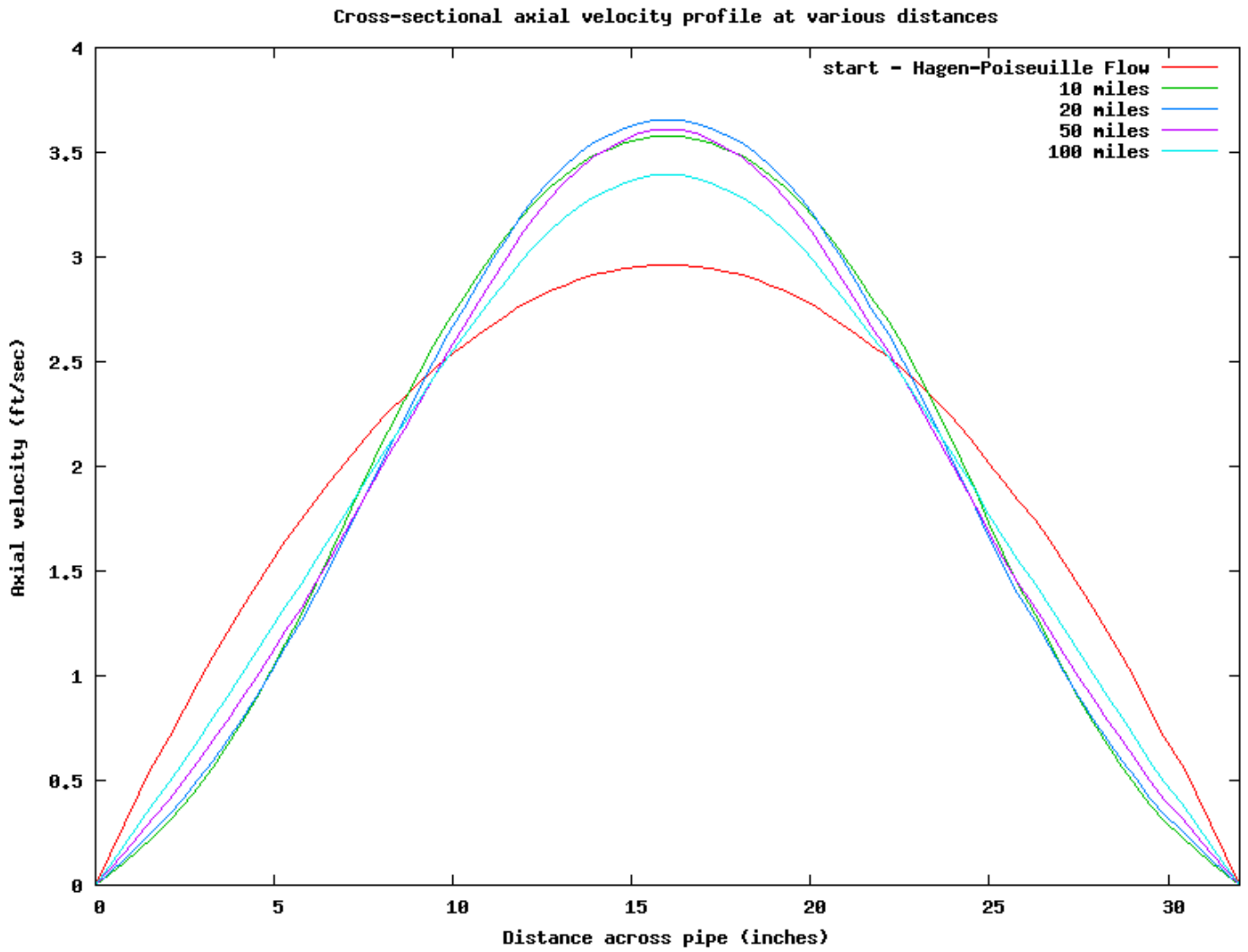
Figure 2 – Temperature along the pipe, averaged over the cross-section

Figure 3 – Axial velocity across the pipe

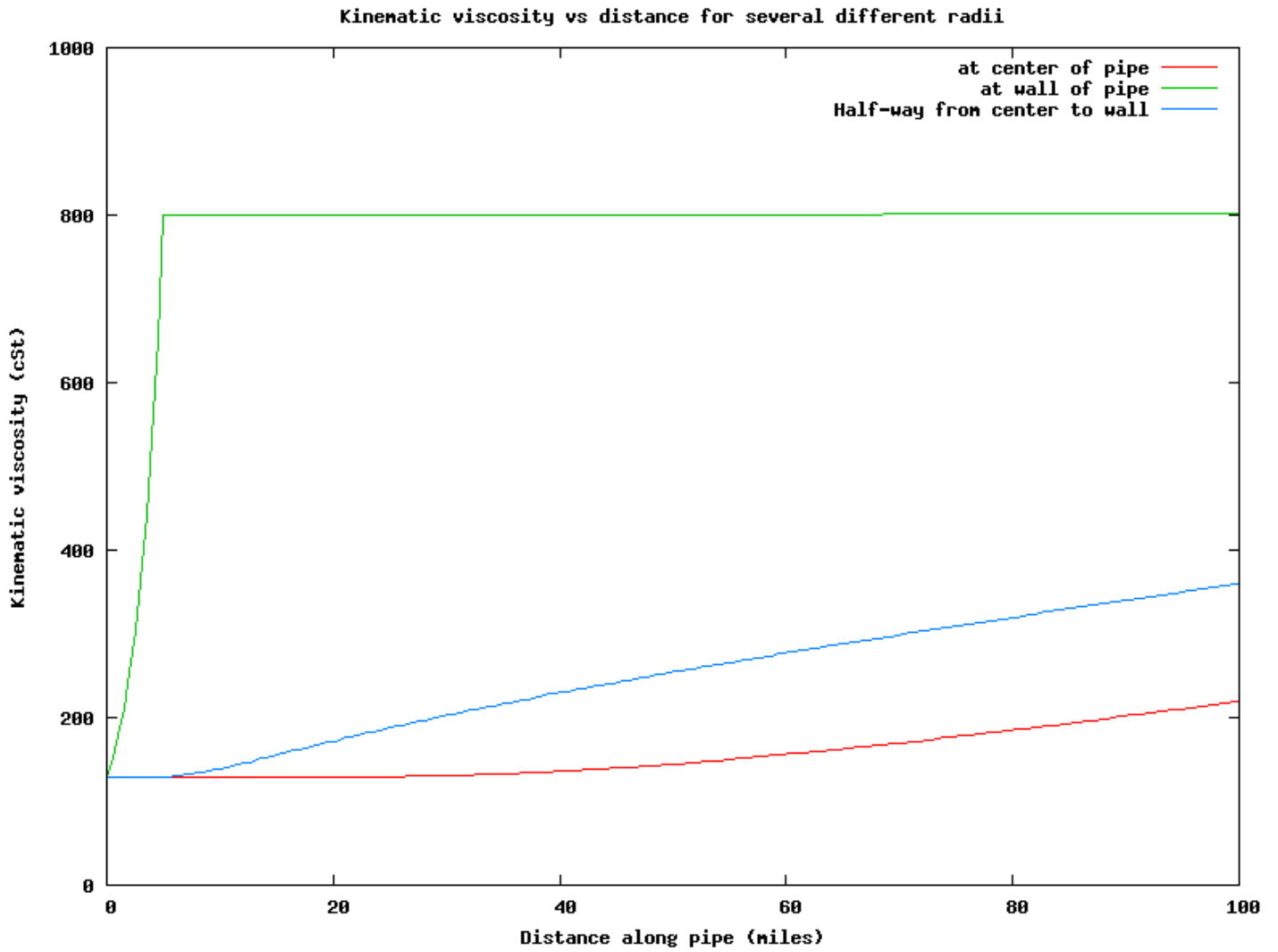
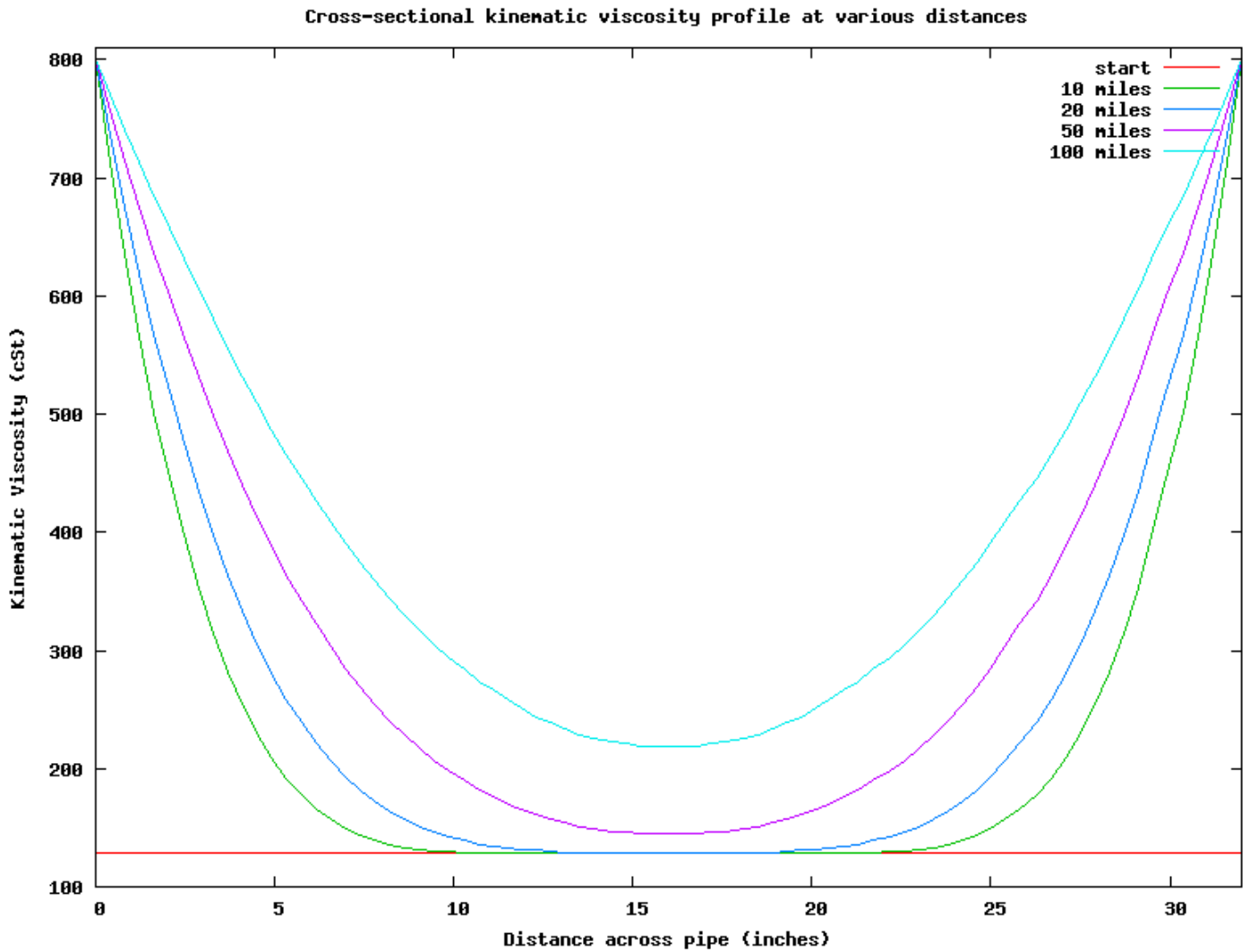


Figure 4 – Dynamic viscosity along the pipe for several radii**Figure 5 – Dynamic viscosity across the pipe**